WE03 ELASTIC POTENTIAL ENERGY

SPH4U

EQUATIONS

- Hooke's Law

$$F_x = \pm kx$$

- Elastic Potential Energy

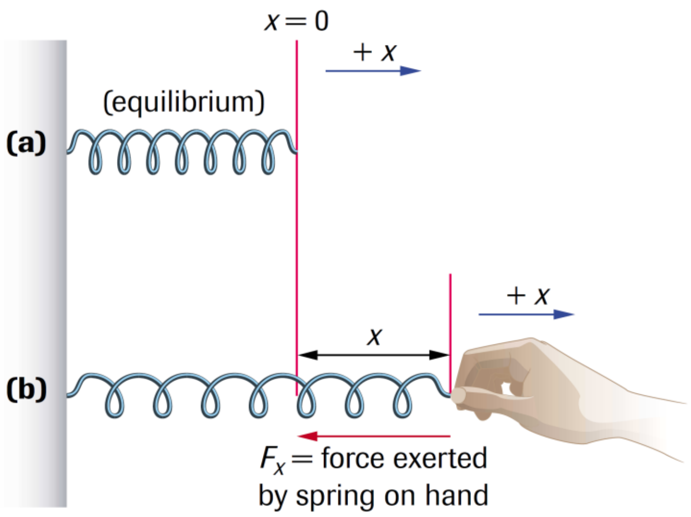
$$E_e = \frac{1}{2}kx^2$$

HOOKE'S LAW

- **Hooke's Law:** the magnitude of the force exerted by a spring is directly proportional to the distance the spring has moved from equilibrium

$$F_x = -kx$$

- **Force Constant (k) [N/m]:** the proportionality constant of a spring
- **Ideal Spring:** a spring that obeys Hooke's law because it experiences no internal or external friction



PROBLEM 1

A student stretches a spring horizontally a distance of 15 mm by applying a force of 0.18 N [E].

(a) Determine the force constant of the spring.

(b) What is the force exerted by the spring on the student?

PROBLEM 1 – SOLUTIONS

(a) $F_x = 0.18\text{ N}$

$x = 15\text{ mm} = 0.015\text{ m}$

$k = ?$

Since the force is applied *to* the spring, we use the equation

$$F_x = kx$$

$$k = \frac{F_x}{x}$$

$$= \frac{0.18\text{ N}}{0.015\text{ m}}$$

$$k = 12\text{ N/m}$$

The force constant is 12 N/m. (Notice the SI units of the force constant.)

(b) According to Newton's third law, if the force applied to the spring is 0.18 N [E], then the force exerted by the spring is 0.18 N [W].

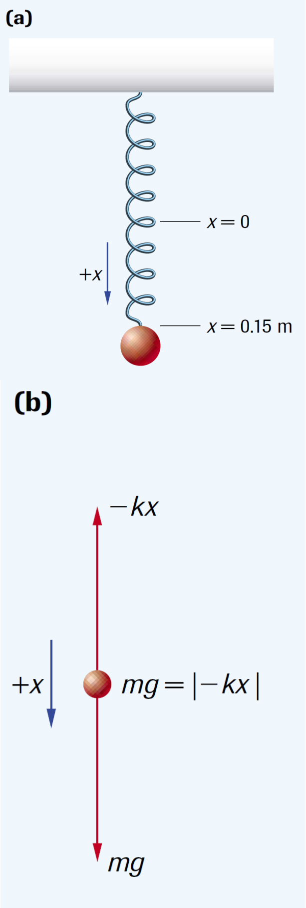
PROBLEM 2

A ball of mass 0.075 kg is hung from a vertical spring that is allowed to stretch slowly from its unstretched equilibrium position until it comes to a new equilibrium position 0.15 m below the initial one. **Figure 3(a)** is a system diagram of the situation, and **Figure 3(b)** is an FBD of the ball at its new equilibrium position.

(a) Determine the force constant of the spring.

(b) If the ball is returned to the spring's unstretched equilibrium position and then allowed to fall, what is the net force on the ball when it has dropped 0.071 m?

(c) Determine the acceleration of the ball at the position specified in (b).



PROBLEM 2 – SOLUTIONS

(a) We measure the extension x of the spring from its original unstretched position ($x = 0$) and choose $+x$ to be downward. Two vertical forces act on the ball: gravity and the upward force of the spring. At the new equilibrium position, the ball is stationary, so the net force acting on it is zero.

$m = 0.075$ kg

$x = 0.15$ m

$k = ?$

$$\sum F_x = 0$$

$$mg + (-kx) = 0$$

$$k = \frac{mg}{x}$$

$$= \frac{(0.075 \text{ kg})(9.8 \text{ N/kg})}{0.15 \text{ m}}$$

$$k = 4.9 \text{ N/m}$$

The force constant is 4.9 N/m.

PROBLEM 2 – SOLUTIONS CONT.

(b) **Figure 4** is the FBD for the ball when $x = 0.071$ m. Considering the components of the forces in the vertical (x) direction:

$$\sum F_x = mg + (-kx)$$

$$= (0.075 \text{ kg})(9.8 \text{ N/kg}) - (4.9 \text{ N/m})(0.071 \text{ m})$$

$$\sum F_x = +0.39 \text{ N}$$

The net force is 0.39 N [down] when the ball has dropped to 0.071 m.

PROBLEM 2 – SOLUTIONS CONT.

(c) $\sum F_y = 0.39\text{ N}$

$a_y = ?$

Applying Newton's second law:

$$\sum F_y = ma_y$$

$$a_y = \frac{\sum F_y}{m}$$

$$= \frac{0.39\text{ N}}{0.075\text{ kg}}$$

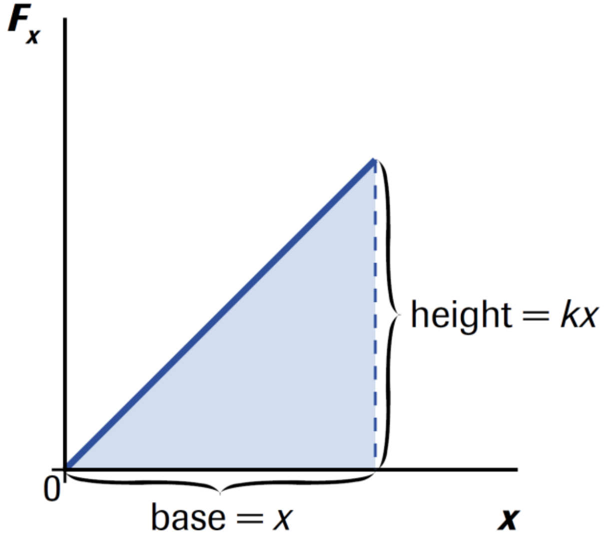
$$a_y = 5.2\text{ m/s}^2$$

The acceleration is 5.2 m/s^2 [down] when the ball is at a spring extension of 0.071 m.

ELASTIC POTENTIAL ENERGY

- **Elastic Potential Energy (E_e) [J]:** the energy stored in an object that is stretched, compressed, bent, or twisted
- To derive, we look at the area graph of force vs displacement

$$W = Fx$$
$$W = \frac{1}{2}x(kx)$$
$$W = E_e = \frac{1}{2}kx^2$$



PROBLEM 3

An apple of mass 0.10 kg is attached to a vertical spring with a force constant of 9.6 N/m. The apple is held so that the spring is at its unstretched equilibrium position, then it is allowed to fall. Neglect the mass of the spring and its kinetic energy.

(a) How much elastic potential energy is stored in the spring when the apple has fallen 11 cm?

(b) What is the speed of the apple when it has fallen 11 cm?

PROBLEM 3 – SOLUTIONS

(a) We measure the extension x of the spring from its original unstretched position ($x = 0$) and choose $+x$ to be downward (**Figure 7**).

$x = 11\text{ cm} = 0.11\text{ m}$

$k = 9.6\text{ N/m}$

$E_e = ?$

$$E_e = \frac{1}{2}kx^2$$

$$= \frac{1}{2}(9.6\text{ N/m})(0.11\text{ m})^2$$

$$E_e = 5.8 \times 10^{-2}\text{ J}$$

The elastic potential energy stored in the spring is 5.8×10^{-2} J.

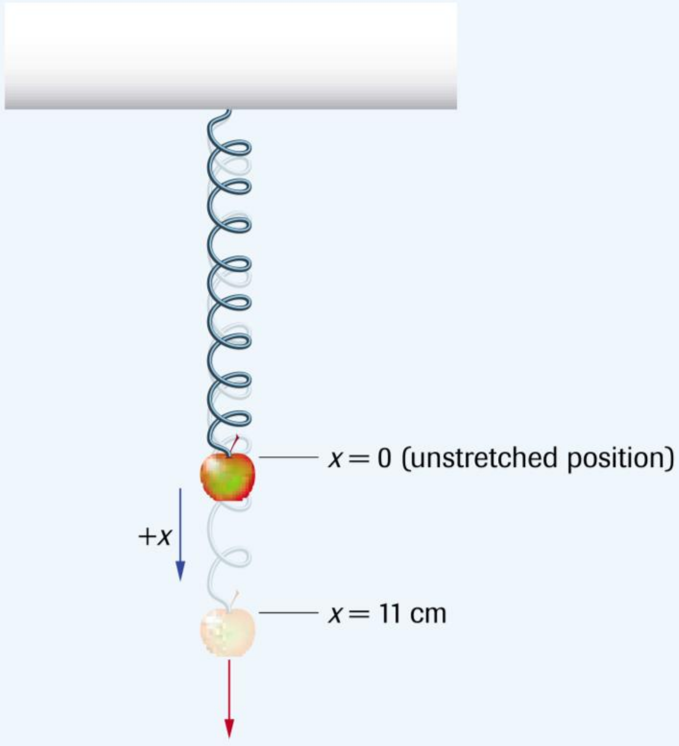


Figure 7

PROBLEM 3 – SOLUTIONS CONT.

(b) We use the prime symbol (′) to represent the final condition of the apple. To apply the law of conservation of energy to determine v', we include the elastic potential energy.

$m = 0.10$ kg

$x = 0.11$ m (for the gravitational potential energy of the apple at the initial position relative to the final position)

$v = 0$

$k = 9.6$ N/m

$g = 9.8$ m/s^2

$x' = 0.11$ m (the extension of the spring when the apple is at the final position)

$E_K = E_e = 0$

$v' = ?$

PROBLEM 3 – SOLUTIONS CONT.

$$
\begin{aligned}
E_T &= E_T' \\
E_g + E_K + E_e &= (E_g + E_K + E_e)' \\
E_g &= (E_K + E_e)' \\
mgx &= \frac{1}{2}mv'^2 + \frac{1}{2}kx'^2 \\
\frac{1}{2}mv'^2 &= mgx - \frac{1}{2}kx'^2 \\
v' &= \pm\sqrt{2gx - \frac{kx'^2}{m}} \\
&= \pm\sqrt{2(9.8\ \text{m/s}^2)(0.11\ \text{m}) - \frac{(9.6\ \text{N/m})(0.11\ \text{m})^2}{0.10\ \text{kg}}} \\
v' &= \pm\ 1.0\ \text{m/s}
\end{aligned}
$$

We choose the positive root because speed is always positive. The speed of the apple is 1.0 m/s.

SUMMARY

- Hooke's law for an ideal spring states that the magnitude of the force exerted by or applied to a spring is directly proportional to the displacement the spring has moved from equilibrium.
- The constant of proportionality k in Hooke's law is the force constant of the spring, measured in newtons per metre.
- Elastic potential energy is the energy stored in objects that are stretched, compressed, twisted, or bent.
- The elastic potential energy stored in a spring is proportional to the force constant of the spring and to the square of the stretch compression.
- The law of conservation of mechanical energy can be applied to a mass–spring system and includes elastic potential energy, kinetic energy, and, in the case of vertical systems, gravitational potential energy.

PRACTICE

Readings

- Section 4.5, pg 203

Questions

- pg 218 #2,5,7,9,10,13